

Bianchi type-VI model with cosmic strings in the presence of a magnetic field

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Abstract

A Bianchi type-VI cosmological model in the presence of a magnetic flux together with a cloud of cosmic strings is considered. In general, the presence of a magnetic field imposes severe restrictions regarding the consistency of the field equations. These difficulties could be overtaken working either in a Bianchi type-VI₀ spacetime or assuming a particular coordinate-dependence of the magnetic field. Using a few plausible assumptions regarding the parametrization of the cosmic strings, some exact analytical solutions are presented. Their asymptotic behavior for large time is exhibited.

Pacs: 95.30.Sf; 98.80.Jk; 04.20.Ha

Key words: Bianchi type-VI model, cosmic string, magnetic field

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1 Introduction

Since the observation of small anisotropies in the microwave background radiation (CMB) [1] and large scale structures [2] it became clear that a pure Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmology could not explain all the properties of our Universe. It is therefore natural to consider anisotropic cosmological models that allow FLRW Universes as special cases.

It is usually assumed that at the very early stages of the evolution of the Universe, during the phase transitions, the symmetry of the Universe was broken spontaneously [3, 4]. Topological defects such as strings, domain walls, monopoles has received considerable attention in cosmology since they could play an important role in the formation of large structure of the Universe.

String cosmological models have been used in attempts to describe the early Universe and to investigate anisotropic dark energy component including a coupling between dark energy and a perfect fluid (dark matter) [5]. Cosmic strings are one dimensional topological defects associated with spontaneous symmetry breaking in gauge theories. Their presence in the early Universe can be justified in the frame of grand unified theories (GUT).

A large number of astrophysical observations proves the existence of magnetic fields in galaxies. Galactic magnetic fields which we observe today could be relics of a coherent magnetic field existing in the early Universe, before galaxy formation. Any theoretical study of cosmological models which contain a magnetic field must take into account that the corresponding Universes are necessarily anisotropic. Among the anisotropic spacetimes, Bianchi type-VI (BVI) space seems to be one of the most convenient for testing different cosmological models.

The object of this paper is to investigate a BVI string cosmological model in the presence of a magnetic field due to an electric current together with the strings. Our paper is organized as follows: In Section 2 we derive the field equations of BVI cosmic string model in the presence of a magnetic field. Section 3 deals with the exact solutions obtained using some simple plausible assumptions and describe their asymptotic behavior. The last Section contains conclusions. In the Appendix A, the geometrical properties and the shear tensor for BVI model are briefly described.

2 Model and field equations

The gravitational field in our case is given by a BVI metric:

$$ds^2 = dt^2 - a_1^2 e^{-2mz} dx^2 - a_2^2 e^{2nz} dy^2 - a_3^2 dz^2, \quad (1)$$

with a_1, a_2, a_3 being the functions of time only. Here m, n are some arbitrary constants and the velocity of light is taken to be unity. It should be emphasized that the BVI metric models a Universe that is anisotropic and space-dependent. The geometrical properties of the BVI spacetime are sketch in Appendix A, including the relationship with other Bianchi-type Universes. The Einstein field equations for BVI metric (1) are written in the form:

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{n^2}{a_3^2} = \kappa T_1^1, \quad (2a)$$

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} - \frac{m^2}{a_3^2} = \kappa T_2^2, \quad (2b)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{mn}{a_3^2} = \kappa T_3^3, \quad (2c)$$

$$\frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} - \frac{m^2 - mn + n^2}{a_3^2} = \kappa T_0^0, \quad (2d)$$

$$m \frac{\dot{a}_1}{a_1} - n \frac{\dot{a}_2}{a_2} - (m - n) \frac{\dot{a}_3}{a_3} = \kappa T_3^0. \quad (2e)$$

Here overdots denote differentiation with respect to time (t). The energy-momentum tensor for a system of cosmic strings and magnetic field is chosen to be

$$T_\mu^\nu = \rho u_\mu u^\nu - \lambda x_\mu x^\nu + E_\mu^\nu, \quad (3)$$

where ρ is the rest energy density of strings with massive particles attached to them and can be expressed as $\rho = \rho_p + \lambda$, where ρ_p is the rest energy density of the particles attached to the strings and λ is the tension density of the system of strings [6, 7, 8] which may be positive or negative. Here u_i is the four velocity and x_i is the direction of the string, obeying the relations

$$u_i u^i = -x_i x^i = 1, \quad u_i x^i = 0. \quad (4)$$

For the electromagnetic field $E_{\mu\nu}$ we adopt the form given by Lichnerowich [9]

$$E_\mu^\nu = \bar{\mu} \left[|h|^2 \left(u_\mu u^\nu - \frac{1}{2} \delta_\mu^\nu \right) - h_\mu h^\nu \right]. \quad (5)$$

Here $\bar{\mu}$ is the magnetic permeability and h_μ is the magnetic flux vector defined by

$$h_\mu = \frac{1}{\bar{\mu}} * F_{\nu\mu} u^\nu, \quad (6)$$

where $*F_{\mu\nu}$ is the dual of the electromagnetic field tensor $F_{\mu\nu}$.

In what follows the comoving coordinates are taken to be $u^0 = 1$, $u^1 = u^2 = u^3 = 0$. We choose the incident magnetic field to be in the direction of z -axis so that the magnetic flux vector has only one nontrivial component, namely $h_3 \neq 0$. In view of the aforementioned assumption from (6) one obtains $F_{23} = F_{31} = 0$. We also assume that the conductivity of the magnetic fluid is infinite which leads to $F_{01} = F_{02} = F_{03} = 0$. Therefore there is only one non-vanishing component of $F_{\mu\nu}$, namely F_{12} . Then from the first set of Maxwell equation

$$F_{\mu\nu;\beta} + F_{\nu\beta;\mu} + F_{\beta\mu;\nu} = 0, \quad (7)$$

where the semicolon stands for covariant derivative, one finds

$$F_{12} = \mathcal{I}, \quad \mathcal{I} = \text{const.} \quad (8)$$

Then from (6) we get

$$h_3 = \frac{a_3 \mathcal{I}}{\bar{\mu} a_1 a_2} \exp[(m - n)z]. \quad (9)$$

Finally, for E_μ^ν one finds the following nontrivial components

$$E_0^0 = -E_1^1 = -E_2^2 = E_3^3 = E = \frac{\mathcal{I}^2}{2\bar{\mu} a_1^2 a_2^2} \exp[2(m - n)z]. \quad (10)$$

Using comoving coordinates we have the following components of energy momentum tensor [11]:

$$T_0^0 - \rho = -T_1^1 = -T_2^2 = T_3^3 - \lambda = \frac{\mathcal{I}^2}{2\bar{\mu} a_1^2 a_2^2} \exp[2(m - n)z]. \quad (11)$$

Taking into account that $T_3^0 = 0$ from (2e) one immediately finds

$$\left(\frac{a_1}{a_3}\right)^m = \mathcal{N} \left(\frac{a_2}{a_3}\right)^n, \quad \mathcal{N} = \text{const.} \quad (12)$$

Let us now introduce a new function

$$v = a_1 a_2 a_3. \quad (13)$$

Then from (12) one finds

$$a_1 = \left(\mathcal{N} v^n a_3^{m-2n} \right)^{1/(m+n)}, \quad (14)$$

$$a_2 = \left(v^m a_3^{n-2m} / \mathcal{N} \right)^{1/(m+n)}. \quad (15)$$

Summation of (2a), (2b), (2c) and three times (2d) gives

$$\frac{\ddot{v}}{v} = 2 \frac{m^2 - mn + n^2}{a_3^2} + \frac{\kappa}{2} \left(3\rho + \lambda + \frac{\mathcal{I}^2 a_3^2}{\bar{\mu} v^2} \exp[2(m-n)z] \right). \quad (16)$$

Let us note that from the energy-momentum conservation law one finds

$$\dot{\rho} + \frac{\dot{v}}{v} \rho - \frac{\dot{a}_3}{a_3} \lambda = 0. \quad (17)$$

Taking into account the z -dependence of the energy-momentum tensor (11) the r. h. s. of eqs. (2) have also a z -dependence, while the metric functions a_i depend on time only. Therefore, in general, eqs. (2) are inconsistent for a BVI model in the presence of a magnetic field. There are two possibilities to restore the consistency of eqs. (2):

1. to limit ourselves to Bianchi type-VI₀ (BVI₀), namely, to consider the case $m = n$;
2. to assume a special z -dependence of the magnetic permeability $\bar{\mu}$ in order to compensate for the z -dependence of the magnetic flux component h_3 , eq. (9). A similar assumption was used by Bali [10] in a different context

In what follows we shall analyze these two possibilities in turn.

3 Solutions of field equations

3.1 BVI₀ model

Assuming $m = n$ we have

$$a_1 = \mathcal{N} a_2, \quad \mathcal{N} = \text{const.} \quad (18)$$

which permit us to express a_1 and a_2 in terms of a_3 and v :

$$a_1 = \mathcal{N}^{1/2} v^{1/2} a_3^{-1/2}, \quad (19)$$

$$a_2 = \mathcal{N}^{-1/2} v^{1/2} a_3^{-1/2}. \quad (20)$$

Consequently Einstein's equations (2) are reduced to the following set of independent equations:

$$\frac{\ddot{a}_3}{a_3} + \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} - \frac{m^2}{a_3^2} = -\frac{\mathcal{K}}{a_1^4}, \quad (21a)$$

$$2\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1^2}{a_1^2} + \frac{m^2}{a_3^2} = \frac{\mathcal{K}}{a_1^4} + \lambda, \quad (21b)$$

$$\frac{\dot{a}_1^2}{a_1^2} + 2\frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{m^2}{a_3^2} = \frac{\mathcal{K}}{a_1^4} + \rho, \quad (21c)$$

where we introduced the notation

$$\mathcal{K} = \frac{\kappa \mathcal{I}^2 \mathcal{N}^2}{2\bar{\mu}}. \quad (22)$$

Therefore there are three equations (21) for four unknown functions a_1 , a_3 , ρ , λ . It is customary to assume a relation between ρ and λ in accordance with the equations of state for strings. The simplest one is a proportionality relation [6]:

$$\rho = \alpha \lambda. \quad (23)$$

The most usual choices of the constant α are [11, 12, 13, 14]

$$\alpha = \begin{cases} 1 & \text{geometric string} \\ 1 + \omega & \omega \geq 0, \quad p \text{ string or Takabayasi string} \\ -1 & \text{Reddy string.} \end{cases} \quad (24)$$

Using relation (23) between ρ and λ we get that

$$\rho v a_3^{-\frac{1}{\alpha}} = C, \quad (25)$$

with C an arbitrary constant.

Now the system of differential equations is determined and we can proceed to solve it. However this system of nonlinear differential equations is quite

intricate and we should resort to numerical simulations which will be reported elsewhere [15].

Here we limit ourselves to investigate the asymptotic behavior of the solutions for large t . For example, we can investigate the possibility to reach an isotropic regime, i.e. all functions a_i to have a similar behavior for $t \rightarrow \infty$

$$a_1 \sim a_2 \sim a_3 \sim v^{\frac{1}{3}}, \quad (26)$$

and consequently for the density of strings

$$\rho \sim v^{\frac{1}{3\alpha}-1}. \quad (27)$$

As it can be observed from (A.11) - (A.13) in the case of isotropic space-time the components of the shear tensor σ_i^i vanish.

To explore this possibility of isotropization, we shall investigate the equation of evolution of v , (16) for $m = n$:

$$\frac{\ddot{v}}{v} = 2\frac{m^2}{a_3^2} + \frac{\kappa}{2}\left(3\rho + \lambda + \frac{\mathcal{I}^2 a_3^2}{\bar{\mu} v^2}\right). \quad (28)$$

Assuming the asymptotic relation between a_i and v (26) we get from (28) the following differential equation valid in the asymptotic regime $t \rightarrow \infty$:

$$\ddot{v} = C_1 v^{\frac{1}{3}} + C_2 v^{\frac{1}{3\alpha}} + C_3 v^{-\frac{1}{3}}, \quad (29)$$

where $C_1 = 2m^2$, $C_2 = \frac{\kappa(3\alpha+1)}{2\alpha}$, $C_3 = \frac{\kappa\mathcal{I}^2}{2\bar{\mu}}$ are some constants. This equation allows the following first integral

$$\int \frac{dv}{\sqrt{C_4 v^{\frac{4}{3}} + C_5 v^{1+\frac{1}{3\alpha}} + C_6 v^{\frac{2}{3}} + C_7}} = t + t_0, \quad (30)$$

where $C_4 = 3C_1/2$, $C_5 = 6\alpha C_2/(3\alpha + 1)$ and $C_6 = 3C_3$. Here t_0 and C_7 are constants of integrations.

We observe that for

$$\frac{1}{3\alpha} \leq \frac{1}{3}, \quad (31)$$

i.e. $\alpha \geq 1$ or $\alpha < 0$, the term with $v^{\frac{4}{3}}$ is dominant in the integration (30) and finally we get

$$v \sim t^3, \quad (32)$$

and consequently

$$a_3 \sim t, \quad (33)$$

and

$$\rho \sim t^{\frac{1}{\alpha}-3}. \quad (34)$$

On the other hand for

$$\frac{1}{3\alpha} > \frac{1}{3}, \quad (35)$$

i.e. $\alpha \in (0, 1)$, the term with $v^{1+\frac{1}{3\alpha}}$ is dominant in the integration (30) and we obtain

$$v \sim t^{\frac{6\alpha}{3\alpha-1}}. \quad (36)$$

For $\alpha \in (\frac{1}{3}, 1)$ we have a power growing in time for v and in the limiting case $\alpha = \frac{1}{3}$ we get an exponential behavior in time. Finally, we note that for $\alpha \in (0, \frac{1}{3})$ there are no solutions in this model presenting an expansion of the Universe for large t .

3.2 BVI model with a specific magnetic permeability

As a second possibility to assure the compatibility of eqs. (2) we assume a special z -dependence of the magnetic permeability [10]:

$$\bar{\mu} = \bar{\mu}_0 \exp[2(m-n)z], \quad (37)$$

with $\bar{\mu}_0$ a constant. Let us note that for $z = 0$ the exponential factor is 1, but for $z \rightarrow \pm\infty$ this factor vanishes or diverges, depending of the sign of the difference $m - n$. This unusual behavior of the magnetic permeability is accepted here as a working hypothesis.

In this case eqs. (2) are compatible and could determine all unknown a_i , ρ and λ . As in the previous case, the numerical simulations will be presented elsewhere [15] and here we shall analyze only the asymptotic behavior of the solutions.

Let us assume that for large t

$$a_3 \sim v^\gamma, \quad (38)$$

and consequently

$$\begin{aligned} a_1 &\sim v^{\frac{n(1-2\gamma)+m\gamma}{m+n}}, \\ a_2 &\sim v^{\frac{n\gamma+m(1-2\gamma)}{m+n}}, \end{aligned} \quad (39)$$

and it is quite simple to verify that all equations (2) support this behavior in the asymptotic regime $t \rightarrow \infty$. For a particular value of γ , namely for $\gamma = \frac{1}{3}$, we recover the isotropization (26) discussed above.

Let us observe that from eq. (17) we have in the asymptotic regime

$$\dot{\rho} = \frac{\dot{v}}{v}(-\rho + \gamma\lambda). \quad (40)$$

We could consider that ρ has an asymptotic behavior correlated with that of v

$$\rho \sim v^\delta, \quad (41)$$

with δ some constant which imply a proportionality relation between ρ and λ as in eq. (23), namely, $\lambda = \frac{1+\delta}{\gamma}\rho$.

With all these assumptions a similar equation of evolution for v as in (30) holds with appropriate constants of integration. The corresponding analysis of the asymptotic behavior of solutions of this equation proceeds as above.

4 Conclusion

We have studied the evolution of an anisotropic Universe given by a BVI cosmological model in presence of a cloud of cosmic strings and magnetic flux. It is found that the system with z -dependent magnetic field within the scope of BVI spacetime is not consistent in general. This difficulty could be overcome working either in a BVI_0 metric, setting $m = n$, or introducing a particular z -dependence of the magnetic permeability.

In a forthcoming paper [15] we shall present some numerical simulations and a detailed analysis of the stability and singularities of the field equations for the present cosmological model.

Acknowledgments

We are thankful to Suresh Kumar for helpful comments. This work is supported in part by a joint Romanian-LIT, JINR, Dubna Research Project, theme no. 05-6-1060-2005/2010. M.V. is partially supported by CNCSIS program IDEI-571/2008 and NUCLEU program PN-09370102, Romania.

Appendix A. BVI cosmological model

The gravitational field in the present paper is given by a BVI cosmological model in the form (1). A suitable choice of m, n as well as the metric functions a_1, a_2, a_3 in the BVI given by (1) evokes the following Bianchi-type Universes:

- for $m = n$ the BVI metric transforms to a BVI₀ one, i.e., $m = n$,
BVI \implies BVI₀ \in open FRW with the line elements

$$ds^2 = dt^2 - a_1^2 e^{-2mz} dx^2 - a_2^2 e^{2mz} dy^2 - a_3^2 dz^2; \quad (\text{A.1})$$

- for $m = -n$ one gets the Bianchi-type V (BV) spacetime;
- for $n = 0$ the BVI metric transforms to a Bianchi-type III (BIII) one, i.e., $n = 0$, BVI \implies BIII with the line elements

$$ds^2 = dt^2 - a_1^2 e^{-2mz} dx^2 - a_2^2 dy^2 - a_3^2 dz^2; \quad (\text{A.2})$$

- for $m = n = 0$ the BVI metric transforms to a Bianchi-type I (BI) one, i.e., $m = n = 0$, BVI \implies BI with the line elements

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 dy^2 - a_3^2 dz^2; \quad (\text{A.3})$$

- for $m = n = 0$ and equal scale factor in all three directions the BVI metric transforms to a Friedmann-Robertson-Walker (FRW) Universe, i.e., $m = n = 0$ and $a = b = c$, BVI \implies FRW with the line elements

$$ds^2 = dt^2 - a^2(dx^2 + dy^2 + dz^2). \quad (\text{A.4})$$

Let us go back to the BVI cosmological model (1). The nontrivial Christoffel symbols of the BVI metric read

$$\begin{aligned} \Gamma_{01}^1 &= \frac{\dot{a}_1}{a_1}, & \Gamma_{02}^2 &= \frac{\dot{a}_2}{a_2}, & \Gamma_{03}^3 &= \frac{\dot{a}_3}{a_3}, \\ \Gamma_{11}^0 &= a_1 \dot{a}_1 e^{-2mz}, & \Gamma_{22}^0 &= a_2 \dot{a}_2 e^{2nz}, & \Gamma_{33}^0 &= a_3 \dot{a}_3, \\ \Gamma_{31}^1 &= -m, & \Gamma_{32}^2 &= n, & \Gamma_{11}^3 &= \frac{m a_1^2}{a_3^2} e^{-2mz}, & \Gamma_{22}^3 &= -\frac{n a_2^2}{a_3^2} e^{2nz}. \end{aligned}$$

The non-vanishing components of Riemann tensor corresponding to (1) are

$$\begin{aligned}
R_{01}^{01} &= -\frac{\ddot{a}_1}{a_1}, \quad R_{02}^{02} = -\frac{\ddot{a}_2}{a_2}, \quad R_{03}^{03} = -\frac{\ddot{a}_3}{a_3}, \\
R_{12}^{12} &= -\frac{mn}{a_3^2} - \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2}, \quad R_{13}^{13} = \frac{m^2}{a_3^2} - \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1}, \quad R_{23}^{23} = \frac{n^2}{a_3^2} - \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3}, \\
R_{31}^{10} &= \frac{m}{a_3^2} \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_3}{a_3} \right), \quad R_{01}^{13} = m \left(\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1} \right), \\
R_{32}^{20} &= \frac{n}{a_3^2} \left(\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1} \right), \quad R_{02}^{23} = n \left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3} \right).
\end{aligned}$$

The nontrivial components of the Ricci tensor are

$$\begin{aligned}
R_3^0 &= -\left(m \frac{\dot{a}_1}{a_1} - n \frac{\dot{a}_2}{a_2} - (m-n) \frac{\dot{a}_3}{a_3} \right), \\
R_0^0 &= \left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} \right), \\
R_1^1 &= \left(\frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} - \frac{m^2 - mn}{a_3^2} \right), \\
R_2^2 &= \left(\frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{n^2 - mn}{a_3^2} \right), \\
R_3^3 &= \left(\frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_3}{a_1 a_3} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} - \frac{m^2 + n^2}{a_3^2} \right).
\end{aligned}$$

The Ricci scalar reads

$$R = 2 \left[\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1 \dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2 \dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3 \dot{a}_1}{a_3 a_1} - \frac{m^2 - mn + n^2}{a_3^2} \right]. \quad (\text{A.5})$$

Let us now find expansion and shear for BVI metric. The expansion is given by

$$\vartheta = u_{;\mu}^\mu = u_\mu^\mu + \Gamma_{\mu\alpha}^\mu u^\alpha, \quad (\text{A.6})$$

and the shear is given by

$$\sigma^2 = \frac{1}{2} \sigma_{\mu\nu} \sigma^{\mu\nu}, \quad (\text{A.7})$$

with

$$\sigma_{\mu\nu} = \frac{1}{2} [u_{\mu;\alpha} P_\nu^\alpha + u_{\nu;\alpha} P_\mu^\alpha] - \frac{1}{3} \vartheta P_{\mu\nu}, \quad (\text{A.8})$$

where the projection vector P :

$$P^2 = P, \quad P_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu, \quad P^\mu_\nu = \delta^\mu_\nu - u^\mu u_\nu. \quad (\text{A.9})$$

In comoving system we have $u^\mu = (1, 0, 0, 0)$. In this case one finds

$$\vartheta = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}, \quad (\text{A.10})$$

and

$$\sigma_{11} = \frac{a_1^2 e^{-2mz}}{3} \left(-2 \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right) \Rightarrow \sigma_1^1 = -\frac{1}{3} \left(-2 \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \right), \quad (\text{A.11})$$

$$\sigma_{22} = \frac{a_2^2 e^{2nz}}{3} \left(-2 \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1} \right) \Rightarrow \sigma_2^2 = -\frac{1}{3} \left(-2 \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1} \right), \quad (\text{A.12})$$

$$\sigma_{33} = \frac{a_3^2}{3} \left(-2 \frac{\dot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} \right) \Rightarrow \sigma_3^3 = -\frac{1}{3} \left(-2 \frac{\dot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} \right). \quad (\text{A.13})$$

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